

# Mars outflow channels: A reappraisal of the estimation of water flow velocities from water depths, regional slopes, and channel floor properties

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[1] Methods used so far to assess the flow velocities of the water commonly assumed to be responsible for forming the major outflow channel systems on Mars have relied widely on various versions of the Manning equation. This has led to problems in allowing for the difference between the accelerations due to gravity on Mars and Earth and for the differences of scale between Martian floods and most river systems on Earth. We reanalyze the problem of estimating water flow velocities in Martian outflow channels using equations based on the Darcy-Weisbach friction factor instead of the Manning  $n$  factor. We give simplified formulae appropriate to Mars for the Darcy-Weisbach friction coefficient as a function of bedrock size distribution. For a given channel floor slope and water flood depth, similar mean flow velocities are implied for a wide range of values of the ratio of bed roughness to water depth relevant to Martian outflow channels. Using a recent rederivation of Manning's equation based on turbulence theory, we obtain a new value of  $0.0545 \text{ s m}^{-1/3}$  for the Manning  $n$  coefficient appropriate to Martian channels and show that previous analyses have generally overestimated (though in some cases underestimated) water flow velocities on Mars by a factor of order two. Combining the consequences of this flow velocity overestimate with likely overestimates of flow depth from assuming bank-full flow, we show that discharges may have been overestimated by a factor of up to 25, leading to corresponding overestimates of subsurface aquifer permeabilities, rates of filling of depressions with water, and grain sizes of sediments on channel floors. Despite the availability of an improved value for the Manning  $n$  coefficient for Mars, we strongly recommend that modified forms of the original version of the Manning equation should be replaced by the modern form or, preferably, by the Darcy-Weisbach equation in future work.

**INDEX TERMS:** 6225 Planetology: Solar System Objects: Mars; 1829 Hydrology: Groundwater hydrology; 1860 Hydrology: Runoff and streamflow; 1815 Hydrology: Erosion and sedimentation; 5415 Planetology: Solid Surface Planets: Erosion and weathering;  
**KEYWORDS:** Mars, outflow channels, water

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## 1. Introduction

[2] The discharge rates and volumes of fluids responsible for carving the Martian outflow channels are of significant importance for understanding both the source of the fluid and its evolution on the surface of Mars. If, as generally accepted (see discussions by Carr [1979] and Baker [2001]), the

channels were formed by release of water from the subsurface, then they carry implications for the volumes and release rates of that water, and hence the availability of water as a liquid, as a function of time on Mars. If, on the other hand, the channels are the result of releases of liquid carbon dioxide or carbon dioxide-water mixtures resulting from decomposition of clathrates [e.g., Milton, 1974; Hoffman, 2000], then there are equally important implications for the volatile content and thermodynamic state of the Martian subsurface. For the present purpose we assume that water is the sole agent

responsible for the erosion of the channels, on the basis of arguments about the thermodynamics of the formation of clathrates [Stewart and Nimmo, 2002] and the behavior of carbon dioxide liquid released into the current Martian surface environment [Wilson and Head, 2002] (though the analysis that follows would be applicable to any low-viscosity liquid).

[3] Most attempts to estimate the discharges (i.e., water volume fluxes) of the Martian outflow channels have employed the empirically derived Manning [1891] equation. This equation expresses the influence of bed roughness and sediment transport in channels through a single parameter, the Manning coefficient,  $n$ , which has the dimensions of time divided by length<sup>1/3</sup>. The Manning equation does not have the simplest functional form that can be derived from dimensional analysis and also (as is the case with the equation due to Chézy [Herschel, 1897]) does not explicitly include the effect of the acceleration due to gravity. Attempts have been made by various authors in the planetary community to allow for the difference between the accelerations due to gravity on Mars and Earth but, even when done correctly, this does not eliminate all problems. In addition to the acceleration due to gravity, the Manning coefficient involves a length scale that has been shown in a recent development by Gioia and Bombardelli [2002] to represent the scale of bed roughness, and thus even when the differing gravities are taken into account, use of the resulting Manning coefficient may not be appropriate on Mars unless the bed roughness in Martian outflow channels is generally similar to that in the terrestrial rivers from which values of the Manning coefficient are derived. Alternative equations describing flow through open and closed conduits have been available for more than a century, and the American Society of Civil Engineers [ASCE Task Force on Friction Factors, 1963] recommends the adoption of the Darcy-Weisbach equation, which utilizes an empirically determined nondimensional friction coefficient, as the optimal approach to understanding flow of water over a wide range of conditions. The nature of this issue for Mars was clearly described by Komar [1979] in an important paper largely ignored by later workers. Here we reiterate the case for the need for a transition in the methods employed by the planetary community to estimate channel discharges: use of the original versions of the Manning and Chézy equations, and of their ad hoc modifications, should preferably be replaced by use of the Darcy-Weisbach equation. If a Manning-type approach is used it should employ the new development by Gioia and Bombardelli [2002], which replaces the original Manning coefficient,  $n$ , with a new, dimensionless coefficient,  $K$ , and explicitly includes gravity and channel floor roughness within the equation.

[4] In order to clarify the issues we first summarize, on the basis of a review by Bathurst [1993], the Darcy-Weisbach equation used to analyze water flow in channels on Earth, employing a dimensionless friction factor to characterize channel bed properties. We then calculate the water flow velocities that would be expected on Mars under various sets of conditions that might plausibly apply to outflow channels. Next we use the recent Gioia and Bombardelli [2002] analysis to derive a value for a dimensionless constant  $K$  which allows Manning coefficients measured under given conditions on Earth to be converted to dimensionless friction factors. We use this constant to derive a first approximation to

the value of Manning's  $n$  coefficient for use on Mars. By comparing the water flow velocities predicted by this new version of Manning's equation with the flow velocities predicted under the same conditions by the Darcy-Weisbach treatment, we optimize the values of both the universal constant  $K$  and the value of  $n$  for use on Mars. We find that the modified versions of the Manning and Chézy equations employed to date by the planetary community generally overestimate, but in some cases underestimate, the maximum water velocity in a given channel by a factor of up to two.

## 2. Basic Relationships

[5] Estimation of the water volume flux  $F$  in a channel requires values for the mean depth of water in the channel,  $d$ , the mean width of the channel,  $W$ , and the sine of the down-flow slope of the channel bed,  $S$ . The volume flux is given by

$$F = dWU_c, \quad (1)$$

where  $U_c$  is the mean water flow velocity in the channel. The requirement is then to express  $U_c$  as a function of  $d$  and  $S$ , taking account of the possible influence of the channel width  $W$ . The latter is incorporated by defining the hydraulic radius of the channel,  $R$ . This is equal to the cross-sectional area of the channel divided by its wetted perimeter, which means that in the case of a channel with sides that are not excessively gently sloping,

$$R = (Wd)/(W + 2d), \quad (2)$$

and so if  $W$  is much greater than  $d$ , as is commonly the case for natural water channels,  $R$  is essentially equal to  $d$ . In the general case therefore we need to specify a relationship giving  $U_c$  as a function of  $R$  and  $S$ .

[6] Most treatments of flow in Martian channels have employed Manning's [1891] equation for this purpose. The original form of the equation is

$$U_c = (R^{2/3}S^{1/2})/n, \quad (3)$$

where  $n$ , the Manning coefficient, has dimensions of time divided by length<sup>1/3</sup>. The functional form of this equation, both in terms of the power of  $R$  and the fact that  $n$  is not dimensionless, contrasts starkly with what is expected from dimensional analysis; furthermore, no explicit account of the effect of the acceleration due to gravity,  $g$ , is included in the equation.

[7] For this reason, it has long been recommended [ASCE Task Force on Friction Factors, 1963] that flow of water in channels should be treated in terms of the Darcy-Weisbach equation [Bathurst, 1993], which relates the mean flow velocity to other parameters via a dimensionless friction factor  $f_c$  such that

$$U_c = [(8gRS)/f_c]^{1/2}. \quad (4)$$

This equation has the correct functional dependence on  $g$ ,  $R$  and  $S$  expected from dimensional analysis [Knudsen and Katz, 1958, pp. 80–81]; it should be noted that some theoretical treatments adopt factors other than 8 in the above expression and thus quote systematically different, but still dimensionless, values for the friction factor. There is a large

body of empirical field and laboratory data giving values for the friction factor  $f_c$  as a function of the nature of the channel bed and the flow conditions, which we now describe.

### 3. Formulae for $f_c$

[8] *Bathurst* [1993] has summarized empirical functions fitting a large body of measurements of the variation of  $f_c$  with bed roughness and water depth for a wide range of flow conditions, and the equations for the first four of the following five flow scenarios are taken directly from his work. In the case of channels with sand beds, *Bathurst* gives implicit expressions for  $f_c$ , and we have manipulated these to yield explicit formulae. The formula for the fifth scenario below is adapted from the one used in the engineering literature for fluid flow in rough pipes [*Knudsen and Katz*, 1958] and is included as a proxy for channels with fixed bed roughness.

#### 3.1. Sand Bed Channels

[9] There are two main regimes for flow in channels where the bed is dominated by sand-size material: a lower regime (corresponding to a plane bed with no transport and having ripples and dunes) and an upper regime (corresponding to a plane bed with transport and having antidunes and chutes and pools). Between the two is a transition zone with bedforms ranging between dunes, plane beds and antidunes. Resistance formulae either take account separately of grain drag and bedform drag or lump the two together. The lumped formulae are

$$\text{Lower regime } R/D_{50} = 0.3724q^{*0.6539}S^{-0.2542}\sigma_g^{0.1050}, \quad (5a)$$

$$\text{Upper regime } R/D_{50} = 0.2836q^{*0.6248}S^{-0.2877}\sigma_g^{0.08013}, \quad (5b)$$

where  $D_{50}$  is the channel bed clast size such that 50% of clasts are smaller than  $D_{50}$ ;  $\sigma_g$  is the geometric standard deviation of the bed clast size distribution (the dimensionless number equal to the ratio of the mean size to the size one standard deviation away from the mean); and  $q^* = q/(g D_{50}^3)^{0.5}$ , where  $q$  is the water volume flux per unit channel width, ( $F/W$ ). Equation (1) shows that  $q = (d U_c)$ , and so these equations can be rewritten as

Lower regime

$$U_c = 4.529d^{-1}(gD_{50}^3)^{0.5}(R/D_{50})^{1.529}S^{0.3888}\sigma_g^{-0.1606}, \quad (6a)$$

Upper regime

$$U_c = 7.515d^{-1}(gD_{50}^3)^{0.5}(R/D_{50})^{1.601}S^{0.4605}\sigma_g^{-0.1283}. \quad (6b)$$

Substituting these expressions for  $U_c$  into equation (4), and recalling that for all practical purposes  $R \approx d$ , leads to

$$\text{Lower regime } (8/f_c)^{1/2} = 4.529(R/D_{50})^{0.02929}S^{-0.1113}\sigma_g^{-0.1606}, \quad (7a)$$

$$\text{Upper regime } (8/f_c)^{1/2} = 7.515(R/D_{50})^{0.1005}S^{-0.03953}\sigma_g^{-0.1283}. \quad (7b)$$

#### 3.2. Gravel Bed Channels

[10] When the channel bed is dominated by gravel-size clasts, the grain size of the bed material is incorporated via

the parameter  $D_{84}$ , the channel bed clast size such that 84% of clasts are smaller than  $D_{84}$ ; account is also taken of irregularities in the depth of the channel by including the maximum channel depth  $d_m$  such that

$$(8/f_c)^{1/2} = 5.75 \log_{10} [(aR)/(3.5D_{84})], \quad (8)$$

where

$$a = 11.1(R/d_m)^{-0.314}. \quad (9)$$

#### 3.3. Boulder Bed Channels

[11] In channels dominated by boulders the relationship is

$$(8/f_c)^{1/2} = 5.62 \log_{10} [R/D_{84}] + 4. \quad (10)$$

#### 3.4. Steep Pool-Fall Channels

[12] These are very steep channels on hillsides where a great deal of transport of coarse material may take place. Although this circumstance is probably not common on Mars, it is included here for completeness. The grain size of the bed material is now incorporated via the parameter  $D_{90}$ , the channel bed clast size such that 90% of clasts are smaller than  $D_{90}$ ; also, since all bed material is assumed to be constantly in motion, the depth parameter used is  $d_s$ , defined as the total depth of water plus sediment. The relationship is

$$(8/f_c)^{1/2} = 5.75 \left\{ 1 - \exp \left[ (-0.05d_s) / (D_{90}S^{1/2}) \right] \right\}^{1/2} \cdot \log_{10} [(8.2d_s)/D_{90}]. \quad (11)$$

#### 3.5. Channels With Fixed Bed Roughness

[13] The following function, taken from the engineering literature [e.g., *Knudsen and Katz*, 1958] for fluid flow in rough tubes, is included to characterize channels in which the roughness elements are fixed so that they cannot be moved by the fluid. This circumstance is probably closest to that which prevails in channels with boulder beds. If  $r$  is the typical size of bed roughness elements,

$$(8/f_c)^{1/2} = 5.657 \log_{10} [R/r] + 6.6303. \quad (12)$$

#### 3.6. Simplifications to the Above Equations

[14] In essence, the equations for  $f_c$  for gravel beds, boulder beds and fixed beds involve only various versions of the ratio of some representation of the channel depth to some measure of the bed roughness scale. In contrast, the equations for sand beds and steep pool-falls also involve the sine of the slope of the channel  $S$ , and additionally the sand bed formula involves the geometric standard deviation of the bed clast size distribution  $\sigma_g$ . However, the dependencies on these extra factors are not strong, as we now discuss. We use data on clast size distributions in Martian channels taken from the Viking [*Golombek and Rapp*, 1997] and

**Table 1.** Values of Grain Size Parameters for Rocks at the Viking 1, Viking 2, and Pathfinder Landing Sites<sup>a</sup>

Lander	$D_{90}/m$	$D_{84}/m$	$D_{50}/m$	$D_{16}/m$	$D_{10}/m$	$\sigma_g$
Viking 1	0.191	0.152	0.058	0.015	0.009	3.3
Viking 2	0.277	0.221	0.084	0.021	0.013	3.3
Pathfinder	0.160	0.120	0.050	0.025	0.020	2.2
Average values	0.209	0.164	0.064	0.020	0.014	2.9

<sup>a</sup>Values are deduced from data of *Golombek and Rapp* [1997] for the two Viking sites and *Golombek et al.* [2003] for Pathfinder.

Pathfinder [*Golombek et al.*, 2003] landing sites, from which we have extracted the grain size distribution parameters summarized in Table 1. We note that these sites are located at the distal ends of outflow channel systems, and that the clasts there now may not be perfectly representative of those transported by the floods because of subsequent wind erosion and denudation. However, we do not think that the present size distributions greatly underestimate the coarser parts of the original distributions representative of the maximum discharge, because the largest clasts present at any given location in a channel must inevitably be deposited by the most energetic phase of the flow. Also, examination of the highest (a few meters) resolution MOC images of channel floors shows that abundant boulders larger than those at the two sites measured are not present at other locations.

[15] In equations (7a) and (7b),  $S$  appears to the powers  $-0.1113$  and  $0.03953$ , respectively. A survey of the Mangala, Athabasca, Ravi and Kasei Valles shows that the bed slopes for Martian outflow channels commonly lie within the range  $S = 1 \times 10^{-3}$  to  $S = 3 \times 10^{-3}$ . The equivalent range of values of  $S^{-0.1113}$  is then 2.16 to 1.91 and the range of values of  $S^{0.03953}$  is 1.31 to 1.26. Equation (7b) seems more likely to be relevant to major floods on Mars than equation (7a), so even if we ignored the detailed effect of slope and fixed  $S^{-0.03953}$  at the average value 1.285 we would incur only a 2% error. Similarly, Table 1 shows that  $\sigma_g$  could plausibly be anywhere in the range 2 to 4, but  $\sigma_g^{0.1283}$  in equation (7b) would then only range from 0.915 to 0.837, a variation of less than 5% around the mean of 0.876.

[16] In equation (8) the term  $a = 11.1 (R/d_m)^{-0.314}$  can be approximated using the observation that in, for example, the Mangala Valles channels a water depth of at least 50 m would mean that the deeper parts of the occupied channels were of order a factor of 2 deeper than the shallower parts. With  $(R/d_m)$  in the plausible range 0.3 to 0.7,  $a$  lies in the range 12.4 to 16.2, a 13% spread around the average of 14.3.

[17] Finally, although we do not consider it relevant to most Martian channels, we consider the term  $\{1 - \exp[(-0.05 d_s)/(D_{90} S^{1/2})]\}^{1/2}$  in equation (11) for steep pool-fall channels. The range of  $S$  relevant to this equation would be  $\sim 0.1$  to nearly 1; meter-sized boulders in water floods at least a few tens of meters deep would imply  $(d_s/D_{90})$  to be within the range 10 to 100. The value of the term would thus lie between  $\sim 0.6$  and  $\sim 1$ , a 25% spread around the average of 0.8.

[18] These results can be used to greatly simplify the equations for  $f_c$ . Equation (7b) for sand bed channels would be  $(8/f_c)^{1/2} = 7.51537 (R/D_{50})^{0.1005} \times 1.285 \times 0.876$ , i.e.,

$$\text{Sand bed } (8/f_c)^{1/2} = 8.46(R/D_{50})^{0.1005}. \quad (13)$$

Equation (8) would become  $(8/f_c)^{1/2} = 5.75 \log_{10} [(14.3 R)/(3.5 D_{84})]$ , which can be written

$$\text{Gravel bed } (8/f_c)^{1/2} = 5.75 \log_{10} (R/D_{84}) + 3.514, \quad (14)$$

and equation (10) would remain as

$$\text{Boulder bed } (8/f_c)^{1/2} = 5.62 \log_{10} (R/D_{84}) + 4.0. \quad (15)$$

Finally, equation (11) would become

$$\text{Steep pool-fall } (8/f_c)^{1/2} = 4.60 \log_{10} (d_s/D_{90}) + 4.203, \quad (16)$$

and equation (12) would remain as

$$\text{Fixed bed } (8/f_c)^{1/2} = 5.657 \log_{10} (R/r) + 6.6303. \quad (17)$$

#### 4. Flow Conditions in Martian Channels

[19] The likely pattern of water flow rates in Martian channels can be illustrated by calculating the water velocity as a function of water depth and bed slope for each of the bed types discussed in section 3. As mentioned in section 3.6, the range of bed slopes found for a number of channels on Mars lies between  $S = 1 \times 10^{-3}$  and  $3 \times 10^{-3}$ . The average of these two values,  $S = 2 \times 10^{-3}$ , is used for the illustration. The range of water depths considered is from 3 m to 300 m. The upper limit is based on the observation that, although some channel systems have total depths of order one km, individual subchannels on the floors of these systems are rarely deeper than a few hundred meters. Several authors have presented values consistent with this [e.g., *Komar*, 1979; *Baker*, 1982; *Robinson and Tanaka*, 1990; *Baker et al.*, 1992; *De Hon and Pani*, 1993; *Carr*, 1996; *Komatsu and Baker*, 1997; *Ori and Mosangini*, 1998; *Williams et al.*, 2000; *Baker*, 2001; *Burr et al.*, 2002; *Chapman et al.*, 2003; *Leask et al.*, 2004], but many of these measurements were made using pre-MOLA topography and so we have examined MOLA profiles through the main parts of the channel systems of the Mangala, Ravi and Athabasca Valles. Table 2 shows the results: the greatest subchannel depth is  $\sim 220$  m and the average of all those measured is  $\sim 100$  m. The lower limit of 3 m used in the following illustration is probably much smaller than the depth of any thermally viable outflow channel on Mars, but is included for comparison with flow rates in terrestrial rivers [see *Baker*, 2001]. In calculations to be presented elsewhere which lead to conclusions broadly similar to those of *Carr* [1983] we find that, after allowing for heat and mass loss from water flowing under typical current Martian environmental conditions, evaporation and freezing prevent floods from traveling more than  $\sim 350$  km if they

**Table 2.** Channel Depth Measurements on Three Outflow Channel Systems

Name of Channel System	Number of Subchannels Measured	Mean Depth of Subchannels, m	Minimum Subchannel Depth, m	Maximum Subchannel Depth, m	Typical Depth of Main Channel, m	Ratio of Main to Subchannel Depths
Mangala	19	115	52	219	500	4.3
Ravi	21	70	21	147	900	12.9
Athabasca	10	61	55	70	150	2.5

are 100 m deep and  $\sim 2100$  km if they are 300 m deep. Thus 30 m water depth might be a more realistic lower limit for channels of interest on Mars. Finally, our survey of the Mangala, Ravi, Athabasca and Kasei channel systems suggests that the total width of the active water-bearing part of the system is typically  $\sim 30$  km. Using these parameters we calculated the water velocity and total volume flux for each of the bed types discussed in section 3. Figure 1a shows the resulting water velocities  $U_c$ , and Figure 1b shows the fluxes  $F$ .

[20] Figure 1a shows that changing the assumptions about the nature of the channel bed makes surprisingly little difference to the water flow velocity found for a given water depth: the largest value is commonly no more than 25% greater than the smallest. This is largely the consequence of the typical water depths on Mars being much greater than the scales of the bed roughness. The same is true of the water fluxes, though the wider range of values that has to be plotted obscures this in Figure 1b. We infer that, for water depths up to 300 m, water flow velocity on Mars rarely exceeded  $\sim 30$  m s $^{-1}$  unless slopes were unusually steep or floods were confined into narrow valleys by preexisting topography. Volume fluxes for a nominal 30 km wide channel system range up to a few times  $10^8$  m $^3$  s $^{-1}$ . These values are similar to those found in numerous analyses reported in the literature and, as we and others have commented elsewhere [e.g., Head *et al.*, 2003], place quite severe demands on the ability of subsurface water systems to deliver water to the surface fast enough.

[21] The Reynolds number  $Re$  for flow in a channel is given by

$$Re = U_c R \rho / \eta, \quad (18)$$

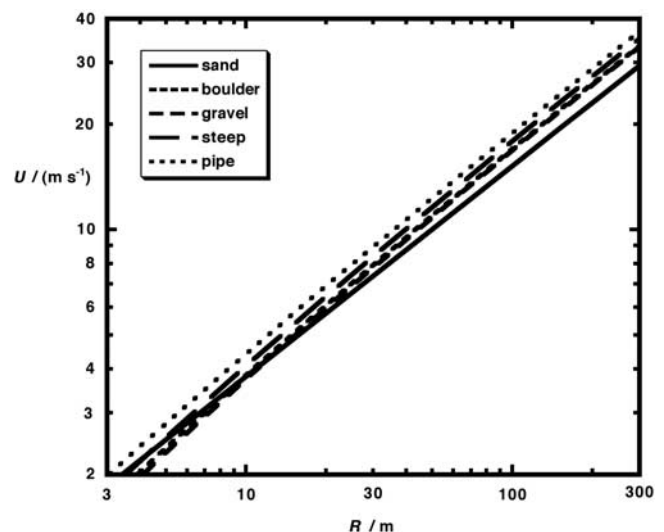
where  $\rho$  and  $\eta$  are the density and viscosity of the water, respectively. Using  $\rho = 1000$  kg m $^{-3}$  and  $\eta = 10^{-3}$  Pa s, the above combinations of water depth and velocity therefore imply that Reynolds numbers for Martian floods ranged up to  $\sim 10^{10}$ . The data summarized by Bathurst [1993] show that values of  $Re$  for terrestrial rivers rarely exceed  $\sim 10^7$ . Thus in calculating flow conditions in deep Martian floods we have tacitly extrapolated the empirical terrestrial data base on which equations (5) to (11) are based by at least three orders of magnitude. Fortunately, values of  $f_c$  are only very weakly dependent on  $Re$  in fully turbulent flows [Knudsen and Katz, 1958], and this extrapolation does not significantly influence our conclusions.

[22] So far, we have tacitly assumed that flow in channels on Mars is subcritical, i.e., that the Froude number  $F_r$  defined by

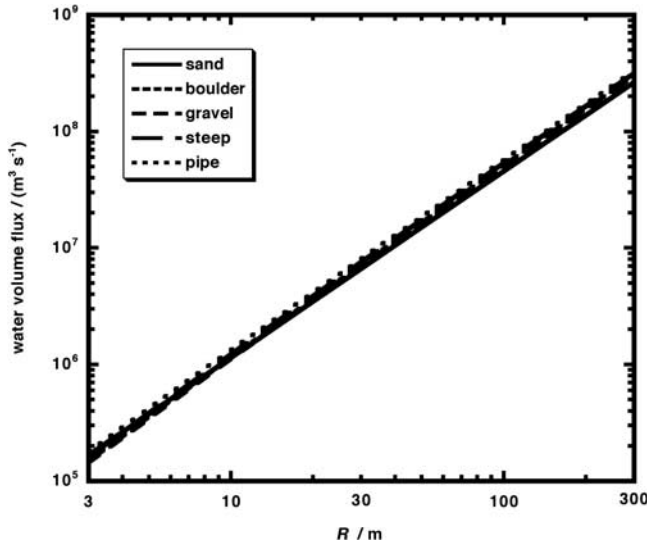
$$F_r = U_c / (gd)^{1/2} \quad (19)$$

is much less than unity. Fluid flows in open channels can only become supercritical, i.e., achieve  $F_r > 1$ , under certain

conditions. Specifically, some kind of constricting nozzle, either dictated by preexisting topography or developed during sediment deposition from the flowing fluid (e.g., to form dunes or anti-dunes), is required [Kieffer, 1989]. However, dynamic interactions between the channel hydraulics and the bed materials, if the latter are sufficiently mobile, appear to prevent the Froude number from exceeding 1 for more than short distances or short periods of time [Grant, 1997]. Therefore there is some doubt as to whether open-channel hydraulic flows may be sustained at supercritical velocities indefinitely. Even if they are not, however, there is still the possibility [Bathurst, 1993] that flow resistance varies with Froude number. This issue does not seem to have been widely explored, though flume experiments by Rosso *et al.* [1990] demonstrate that neglecting the Froude number dependence of the friction factor introduces only a 5–10% error in determining resistance in channels with gradients less than 0.05. Given the gravity difference, this would correspond to gradients less than  $[9.8/3.74] \times 0.05 = 0.13$  on Mars, easily satisfied by the majority of Martian channels. To investigate this issue further we make the assumption, justified earlier, that for the channels of interest here  $d$  in equation (19) is essentially equal to  $R$ . Then, keeping our illustrative channel slope  $S = 0.002$ , for every value of  $R$  for which we evaluated  $f_c$  and hence  $U_c$  in Figure 1a we can also



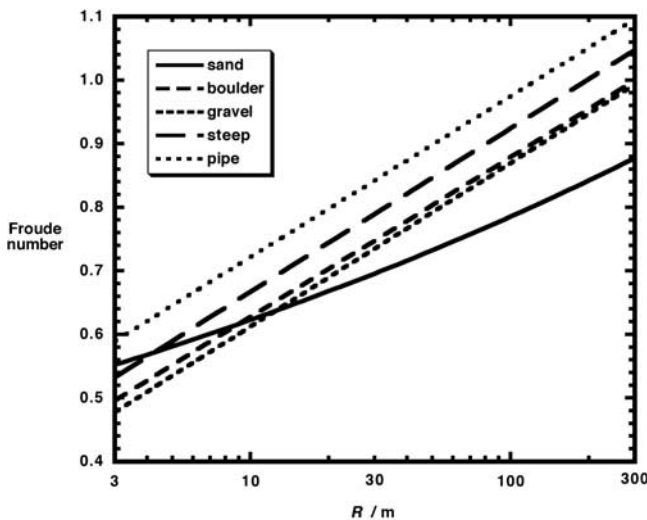
**Figure 1a.** Variation of water flow velocity  $U_c$  with water depth  $R$  for 5 different types of bed roughness in a channel having a bed slope of  $S = 2 \times 10^{-3}$ ; “sand,” “boulder,” and “gravel” refer to the typical transportable bed material grain size; “steep” refers to pool and fall channels on steep hillsides; and “pipe” refers to a bed with fixed roughness such as that in a pipe.



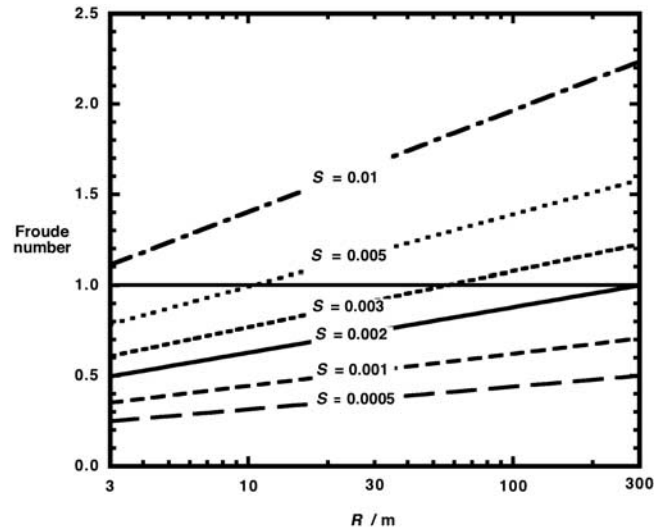
**Figure 1b.** Values of water volume flux as a function of water depth  $R$  in 30 km wide channels having the same bed properties as those listed for Figure 1a.

determine  $F_r$ . The results are plotted in Figure 2a and show that, for most Martian outflow channels, critical and supercritical flow will only occur for water depths greater than a value that varies from 100 m to more than 300 m, depending on bed material particle size. For steeper channels, critical flow will be approached for smaller water depths, and this is illustrated in Figure 2b, where the Froude number is plotted as a function of water depth for the boulder bed friction factor and a wide range of slopes on Mars. We stress again that critical to supercritical flow is not likely to occur other than at local constrictions for most channels.

[23] We now turn to the issue of how Manning's equation can be related to the Darcy-Weisbach treatment in section 3



**Figure 2a.** Variation of Froude number  $F_r$  with water depth  $R$  in channels having the same bed properties as those listed for Figure 1a.



**Figure 2b.** Variation of Froude number  $F_r$  with water depth  $R$  for channels having beds dominated by boulders for various mean channel floor slopes. The lines are labeled with the floor slope  $S$ , and the horizontal line at  $F_r = 1$  indicates critical flow.

and what value of  $n$  might reasonably be employed if the Manning equation is applied to Martian outflow channels (which we do not recommend, however).

## 5. Modified Manning Equation and Values of $n$ for Mars

[24] *Gioia and Bombardelli* [2002] have recently shown that an analogue of Manning's equation can be derived from turbulence theory in a way that replaces Manning's  $n$  with a dimensionless constant  $K$  and also deals with the unexpected power of  $R$ . In terms of the notation used here they give

$$U_c = K(R/r)^{1/6}(RgS)^{1/2}, \quad (20)$$

where  $K$  is a dimensionless constant and  $r$  is the typical size of the (assumed monodisperse) bed roughness elements. The overall power of  $R$  is  $[(1/2) + (1/6) =] 2/3$ , as in the original version of Manning's equation, and Manning's coefficient  $n$  is seen to be a compound of the dimensionless constant  $K$ , the bed roughness scale  $r$  to the power  $1/6$ , and the acceleration due to gravity. The relationship is

$$K = r^{1/6}g^{-1/2}n^{-1}. \quad (21)$$

[25] *Gioia and Bombardelli* [2002] do not give values for the constant  $K$ , but it can be evaluated using terrestrial river data on how the Manning  $n$  value varies as a function of  $r$ . The  $(r, n)$  values in Table 3 are derived from Table 4.1 in *Bathurst* [1993], which gives ranges of values of  $r$  for various channel bed types together with the corresponding ranges of values of  $n$ . Equation (21) is used to find the implied value of  $K$  for each  $(r, n)$  pair (using, of course, the value of  $g = 9.8 \text{ m s}^{-2}$  appropriate to the Earth, where the data were measured). The arithmetic mean of the

**Table 3.** Values of Channel Hydraulic Radius  $r$ , Corresponding Manning Coefficient  $n$ , and Implied Gioia and Bombardelli Constant  $K$  for Various Channel Bed Types<sup>a</sup>

Bed Type	Sand	Gravel	Boulder
Range of $r$	1–2 mm	10–100 mm	0.1–0.2 m
Range of $n$	0.01–0.04 s m <sup>-1/3</sup>	0.02–0.07 s m <sup>-1/3</sup>	0.03–0.2 s m <sup>-1/3</sup>
Range of $K$	10.10–2.83	7.41–3.11	7.25–1.22

<sup>a</sup>The values of  $r$  and  $n$  are slightly modified from Table 4.1 of *Bathurst* [1993]. The implied value of  $K$  is obtained from equation (14) of *Gioia and Bombardelli* [2002].

6 values of  $K$  given in Table 3 is close to 5.32 and for the moment we adopt this as our interim best estimate.

[26] We can now use the Martian rock size distributions in Table 1 (tacitly assuming that they are relevant to other locations on Mars) together with the above-derived estimate of  $K$ , to evaluate the most appropriate values of  $n$  for use in Martian outflow channels. To do this we invert equation (21) to give

$$n = r^{1/6} g^{-1/2} K^{-1}. \quad (22)$$

We have a value for  $K = 5.32$  and can insert  $g = 3.74 \text{ m s}^{-2}$  for Mars, but must decide on a value to use for  $r$ . *Gioia and Bombardelli* [2002] define  $r$  as the “typical size” of the bed roughness elements and so the median rock size,  $D_{50}$ , from Table 1 is used, implying  $r = \sim 0.064 \text{ m}$ . The corresponding value of  $n$  is  $0.061 \text{ s m}^{-1/3}$ ; however, the uncertainty in this value, taking account of the spread of values of  $K$  in Table 3, is at least 30%, so it would be more appropriate to say that  $n$  probably lies between 0.04 and  $0.08 \text{ s m}^{-1/3}$ .

[27] In order to try to improve the best estimate of  $n$ , we have used equation (20) to calculate the water flow velocity as a function of water depth for a wide range of values of  $n$ , using the same bed slope that was used in the illustrations in section 4 using the Darcy-Weisbach formulae. The ratio,  $\lambda$ , of the Manning-derived flow velocity to the average of the Darcy-Weisbach flow velocities is plotted as a function of  $n$  in Figure 3, and the value of  $n$  corresponding to  $\lambda = 1$  is found to be  $n = 0.0545 \text{ s m}^{-1/3}$ . The corresponding value of the *Gioia and Bombardelli* [2002]  $K$  factor is then 6.01, well within the range found in Table 3.

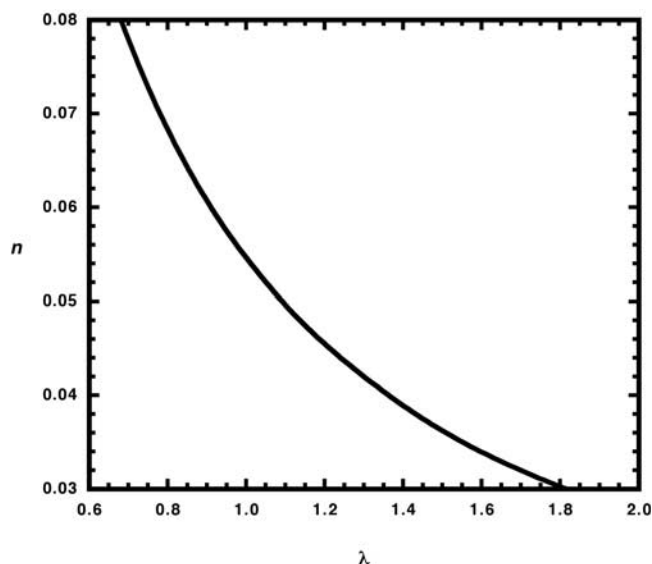
[28] Our best estimate of  $n = 0.0545 \text{ s m}^{-1/3}$  is compared in Table 4 with the values of  $n$  used in most of the work published so far on discharges in Martian channel systems. Many of the values used by the earlier authors are significantly smaller than the best estimate found here, though later workers have tended to use larger values or a wide range of values. We infer that water flow velocities on Mars, and hence water volume fluxes, were overestimated by a factor of order 2 in much of the early work, and that some more recent studies both overestimate and underestimate flow velocities by up to a similar factor.

## 6. Implications

[29] Errors in estimates of water flow velocities and volume fluxes in Martian outflow channels have a bearing on various other quantities deduced from them. For example, water volume fluxes have been used to infer the permeabilities of subsurface aquifers feeding water outbreaks [*Tanaka and Chapman*, 1990; *Zimbelman et al.*, 1992; *Head et al.*, 2003; *Manga*, 2004] and to estimate the

time required to fill basins, including the northern lowlands of Mars, that may have contained long-lived water bodies [*Baker et al.*, 1991; *Williams et al.*, 2000; *Kreslavsky and Head*, 2003; *Carr and Head*, 2003; *Wilson and Head*, 2003]; and water velocities have been used to infer sediment characteristics from observed bedforms or other deposit characteristics on channel floors [e.g., *Ori and Mosangini*, 1998; *Ori et al.*, 2000; *Chapman et al.*, 2003; *Burr et al.*, 2002].

[30] The analysis presented so far demonstrates that flow velocities have commonly been overestimated by a factor of 2, in large part due to use of the Manning equation. If no other factors were involved, this would imply that volume fluxes had been overestimated by a similar factor. However, many investigations have assumed that major channel systems were bank-full for much of the duration of water flow through them. We assert that this is very unlikely. If, as seems incontrovertible in most cases, at least a large fraction of the depth of an outflow channel is due to erosion of the preexisting surface by the flood in the channel, then the only way that a channel can be bank-full for most of the period of its formation is for the water volume flux to increase with time to compensate for the increasing depth. Furthermore, if the bed erosion rate is proportional to the energy in the flow, i.e., to the water velocity squared, which is in turn proportional to the flow depth (equation (4)), then the volume flux



**Figure 3.** Variation of the Manning coefficient,  $n$ , with the ratio,  $\lambda$ , of the water flow velocity derived from Manning’s equation to the water flow velocity derived from the Darcy-Weisbach equation. The optimum value of  $n$  for Mars,  $0.0545 \text{ s m}^{-1/3}$ , corresponds to  $\lambda = 1$ .

**Table 4.** Comparison of Values of the Manning Coefficient  $n$  Used for Mars by Various Authors With the Value Found in This Work

Source Reference	Value(s) Used for $n$	Difference From This Work <sup>a</sup>	% Difference From This Work <sup>a</sup>
This work	0.0545		
<i>Carr</i> [1979]	0.030	-0.0245	-45
<i>Carr</i> [1996]	0.040	-0.0145	-27
<i>Baker</i> [1982]	0.040	-0.0145	-27
<i>Robinson and Tanaka</i> [1990]	0.015 – 0.035	-0.0195 to -0.0395	-72 to -33
<i>De Hon and Pani</i> [1993]	0.067 – 0.133	0.0125 to 0.0785	23 to 144
<i>Williams et al.</i> [2000]	0.010 – 0.070	-0.0155 to 0.0445	-28 to 82
<i>Burr et al.</i> [2002]	0.040	-0.0145	-27
<i>Chapman et al.</i> [2003]	0.030	-0.0245	-45
<i>Kereszturi</i> [2003]	0.075	-0.0205	-38

<sup>a</sup>Positive values imply other authors' coefficients are larger than found here and vice versa.

would need to increase exponentially with time to maintain bank-full flow. This is grossly at variance with what might be expected. Whether due to the draining of an aquifer, as seems to be common on Mars, or the draining of a dammed lake, as in many floods on Earth, the most likely pattern of activity is a decrease in discharge with time as the pressure gradient driving the flow decreases. Thus while we concede that many outflow events on Mars may begin with a transient period of very high water flux, during which patterns of bed erosion are established [e.g., *Schumm et al.*, 1996], we consider that the bulk of the water released is largely confined to subchannels within the main channel, and that even these cannot have been bank-full for all of the time of their formation.

[31] Table 2 shows that the bank-full depths of the subchannels that we measured are less than those of the main channel systems by factors of 4.3, 12.8 and 2.5, respectively. Despite the large spread and small sample size, these measurements suggest that reliance on a bank-full assumption has caused water depths to be overestimated by a factor that is commonly of order 5 to 6. Equation (4) then implies that flow velocities have similarly been overestimated by a factor of order  $5^{1/2}$  to  $6^{1/2} = \sim 2.2$  to 2.4 as a result of this factor alone and, combined with the effect of the factor of 2 due to using an inappropriate Manning coefficient, the aggregate result is an overestimate of water velocities by a factor which is also close to 5. Equation (1) then shows that the combination of these effects leads to an overestimate of volume fluxes by a factor of  $\sim 25$ .

[32] An error of this magnitude has several important consequences. For example, it would cause an overestimate, by the volume flux factor of 25, of the permeability implied for an aquifer system supplying water to an outflow channel, and would lead to an underestimate by the same factor of the time required for the water to fill a depression of a given size. It would also lead to an overestimate of the grain size of the sedimentary material forming structures on a channel floor by a factor which, assuming turbulent clast-fluid relative motion, would be equal to the square of the water velocity factor, i.e.,  $\sim 5^2$ , also equal to 25.

## 7. Summary

[33] 1. We have presented arguments for abandoning the use of the original form of Manning's equation in

the analysis of planetary water flows and using instead the Darcy-Weisbach equation. By utilizing a recent development by *Gioia and Bombardelli* [2002] we have quantified the deficiencies in the attempts of many previous workers to modify Manning's equation for application to Mars, thus extending the insightful analysis of *Komar* [1979]. We find that many previous workers have overestimated (and in some cases underestimated) water flow velocities on Mars by a factor of order 2 due to use of an inappropriate Manning coefficient (see Table 4).

[34] 2. For future authors who insist on using the original version of Manning's equation for water flows on Mars we have derived an optimum value of Manning's coefficient  $n$  equal to  $0.0545 \text{ s m}^{-1/3}$ . As part of this analysis we used terrestrial river data to derive the value 6.01 for the empirical dimensionless constant  $K$  in *Gioia and Bombardelli's* [2002] modified version of Manning's equation which, if the Darcy-Weisbach equation is not used, is greatly to be preferred over the original version.

[35] 3. We have used the rock size distributions at the Viking and Pathfinder landing sites to derive some of the near-constant parameters for Mars in the expressions for the friction factor  $f_c$  used in the Darcy-Weisbach equation. This will simplify the use of this equation in future investigations when high-resolution images provide rock size distributions on the floors of Martian outflow channels. Clearly measurements of these distributions should be a key target for the HiRISE instrument on Mars Reconnaissance Orbiter.

[36] 4. We have briefly discussed the issue of critical or supercritical flow in Martian channels and shown that it is somewhat less likely to occur than on Earth.

[37] 5. We have presented arguments implying that the major outflow channel systems on Mars were not bank-full for much of the duration of their formation and that most of the water was confined to subchannels on the floors of the major channels. As a result, we infer that water velocities have commonly been overestimated by a factor of order 5 and that volume discharge rates have been overestimated by a factor of order 25 in those cases where bank-full flow was assumed. The potential consequences are overestimates by a factor of  $\sim 25$  in the permeabilities of aquifer systems supplying water to outflow events, underestimates by a factor of  $\sim 25$  in the time required for the floods to fill depressions, and overestimates by a factor of  $\sim 25$  of the typical grain



sizes of sedimentary materials forming structures on channel floors. Despite all of these considerations, the Martian outflow channel events were still gargantuan by any terrestrial standards, being nearly two orders of magnitude larger than the greatest floods on Earth in terms of volume discharge rate.

## Notation

$D_{50}$	50% of channel bed clasts are smaller than this size, m.
$D_{84}$	84% of channel bed clasts are smaller than this size, m.
$D_{90}$	90% of channel bed clasts are smaller than this size, m.
$F$	volume flux of water in channel, $\text{m}^3 \text{s}^{-1}$ .
$F_r$	Froude number, dimensionless.
$K$	constant relating $r$ and $n$ , dimensionless.
$R$	hydraulic radius of channel, m.
$S$	sine of channel bed slope, dimensionless.
$U_c$	mean flow velocity of water through channel, $\text{m s}^{-1}$ .
$W$	mean width of channel, m.
$d$	mean depth of channel, m.
$d_s$	depth of water plus sediment in channel, m.
$d_m$	maximum depth of channel, m.
$f_c$	Darcy-Weisbach friction factor for channel, dimensionless.
$g$	acceleration due to gravity, $\text{m s}^{-2}$ .
$n$	Manning roughness coefficient, $\text{s m}^{-1/3}$ .
$q$	water volume flux per unit channel width, $\text{m}^2 \text{s}^{-1}$ .
$r$	typical roughness scale of fixed bed, m.
$\eta$	water viscosity, Pa s.
$\rho$	water density, $\text{kg m}^{-3}$ .
$\sigma_g$	geometric standard deviation of bed clast size distribution, dimensionless.

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